Exercises for seminar week VII, November 2 - 5, ECON3215/4215, fall 2010

A farmer has at his disposal 100 hectares of land. (A hectare, abbreviated ha, is 10,000 square meters.) The land may be used for growing wheat or barley. The wheat yield depends on the weather (such as the occurrence of frost), while the barley yield does not. If the weather is good the farmer receives a yield on wheat land worth NOK 4500 per ha, whereas if the weather is bad the value of the wheat crop is NOK 1500 per ha only. The probability of bad weather is 1/3. The yield on barley is NOK 2500 per ha in any case. Irrespective of the choice of crop, the cost of fertilizers, seeds *et cetera* is NOK 1000 per ha.

(i)   Assume the farmer choose to produce wheat only. Show that in this case his expected net income equals NOK 250,000.

(ii)   If the farmer only considers the two extremes, "all barley" or "all wheat", should he then choose all wheat? Explain!

(iii)   Assume that the farmer has preferences that satisfy the Expected Utility Theorem and that his (von Neuman-Morgenstern) utility function is $u(x)$, where x is net income. What are the expressions for the farmer's expected utility in the two cases considered in (ii)?

(iv)   Assume now that the farmer may choose a mix between wheat and barley. Let Y denote the number of hectares of land used for barley. Net income in good weather conditions is denoted x_G while net income in bad weather conditions is x_B. Show that

$$x_G = 350{,}000 - 2{,}000\ Y$$

$$x_B = 50{,}000 + 1{,}000\ Y$$

(v)   Derive the expression for the farmer's expected utility and show how it depends on Y. Explain how we may (in principle) find the optimal value of Y.

(vi)   What is the optimal value of Y if the utility function is the natural logarithm, that is, $u(x) = \log x$.

The farmer described above lives in the East of the country. There is another farmer, living in the West, who is equal to the first one in all relevant aspects (farm size, income for the various crops under different weather conditions, costs). The probability of bad weather is the same in the West as in the East, namely 1/3, and these events are independent. Both farmers have the utility function $u(x) = \log x$.

(vii)   Assume that both farmers use Y hectares of land for barley. Before the growing season, they can trade in contingent goods. What are the contingent goods in this case? Describe the initial ownership structure.

(viii) Find the competitive equilibrium in the market for contingent goods described in (vii).

(ix)   Suppose that the two farmers make a joint decision about Y. Formulate the expression they would want to maximize. Will the optimal value of Y be smaller than, equal to or greater than the solution to (vi)?